Humans vs. Zombies: A Mathematical Model

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The Goal

The goal of this project is to find a realistic model in which humans not only survive a zombie outbreak but return to and exceed prior population levels.

Assumptions

In this mathematical model, several key assumptions are made, drawing inspiration from a combination of sources including the film I Am Legend, popular cultural interpretations, and my own ideas. The assumptions are as follows:

- Zombies are infected with a strange form of cancer which does not kill the host but feeds on organic material obtained from humans (brains). A side effect is extreme aggression and the desire to spread the disease when not in need of feeding.
- When humans and zombies interact, one of three results are possible. The zombie infects the human, (turning them into a zombie), the zombie kills the human (to eat or for sport), or the human kills the zombie.
- Because dead zombies and humans can return in zombie form, the humans burn the bodies of humans and the zombies to prevent further outbreak.
- Humans are born and die at the same rate in this model as they do in real life, and there are the same number of people in the model as there are in the real world.
- A very small portion of the human population is immune (I) to zombification (0.1%)
- The rest of the human population is susceptible (S) to zombification (99.9%)
- The outbreak occurs as a result of a newfound cure for cancer. The cure for cancer is administered at birth and cannot be given to adults. Unexpectedly, the cure for cancer has side effects. 99% are successfully made immune to cancer (and zombification), but 0.5% die from complications with the vaccination and another 0.5% are turned into zombies.
- Despite this unexpected and catastrophic outcome, the humans persist in applying the antidote to newborns because of the significantly larger percentage of success.

The Model

I’ = -(Natural death rate*I) - (Death rate from zombie attacks*I*Z) + (Vaccination success rate*birth rate*(I+S))

S’ = -(Natural death rate*S) – (Death rate from zombie attacks*S*Z) – (rate of zombification*S*Z)

Z’ = (rate of zombification*S*Z) – (Zombie death rate from humans*(I+S)*Z) + (zombification rate from vaccination*rate of birth*(I+S))

D’ = (natural death rate*I) + (natural death rate*S) + (death from zombies rate*I*Z) + (death from zombies rate*S*Z) + (Zombie death rate from humans*I*S*Z) + (rate of death from vaccination*rate of birth*(I+S))

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Where:

(I) Is the immune population

(S) is the susceptible population

(Z) is the zombie population

(D) is the number of deaths

Solving the System of Equations

Using a Runge-Kutta-4 n-dimensional code in Matlab, the previous 4 equations were solved and plotted. (see code in appendix)

Further assumptions for the values of each variable had to be made. Note that slightly different values of variables led to vastly different outcomes. (see below for assumptions and explanations.) Also note that the constants were approximated in such a way the time units would be worth 1 day each. 100,000 time steps were taken for precision accuracy.

bR = .00004781; setting birth rate (there are 360,000 births per day, so 360,000/7.53b per person per day)
Nd = .00002013; setting rate of natural deaths (151,600 deaths per day so 151k/7.53b per person per day)
Zd = .02; setting rate of deaths from zombies (it seems reasonable that most of the time, humans are able to escape or kill the zombie, though they may be infected)
Zb = .18; setting rate of zombification
Q = .8; rate of humans killing zombies, it seems reasonable that humans should succeed far more often than the zombies given our advanced weaponry, militaries and intelligence.
Im = .99; Ki = .005; Zo = .005 (vaccination outcomes)

The result was the following graph, where green represents immune humans, blue represents susceptible humans, and red represents zombies. The x axis represents time in days, the y axis shows the population, in billions.

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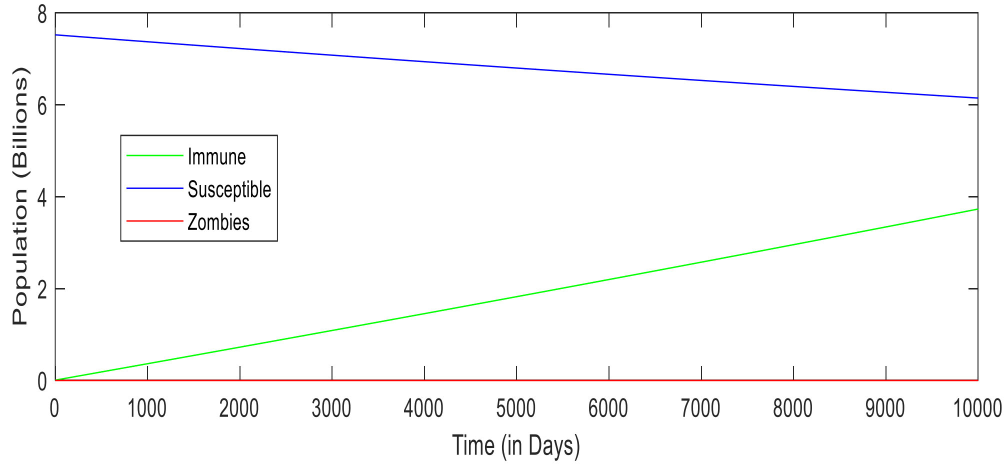
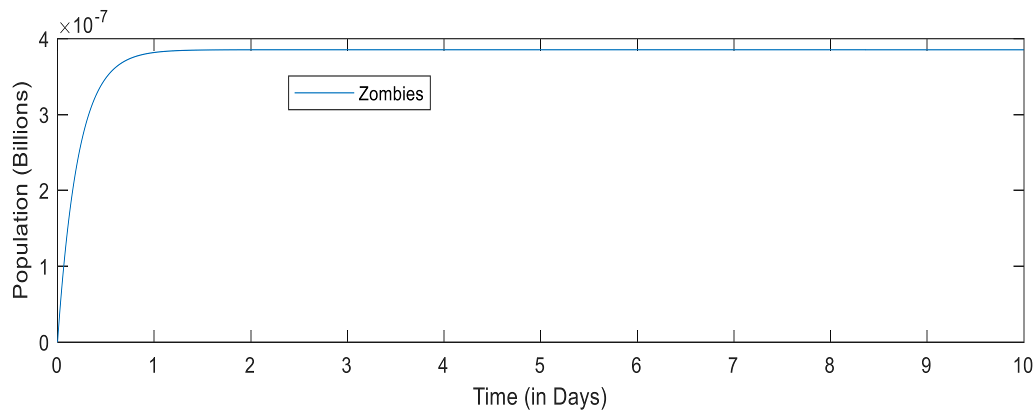


As can be seen from the graph, the humans emerge victorious over the zombie threat. For a closer look at how the zombie population changed over time, see the below graph and analysis. Again, the x axis represents time in days and the y axis represents the population, in billions.



As can be seen, the zombie population initially grows exponentially quickly, to roughly 380 (3.8 billion*10^-7), before slowing in growth and reaching equilibrium. The zombies initially grow extremely quickly due to the abundance of available targets, but as their number increases, the human's effectiveness in killing them takes command and damps the population from growing beyond approximately 380. Note that this equilibrium will continue, as newborns continually have a 0.5% chance of becoming zombies at the time of vaccination. This poses the interesting scenario in which zombies and humans would coexist without one bringing the demise of the other. Overall, however, the humans are largely successful and continue as a species without much trouble. The above scenario seems most reasonable.

Alternative Scenarios

In this section, I explore other interesting outcomes which result from a variation of the values of the variables.

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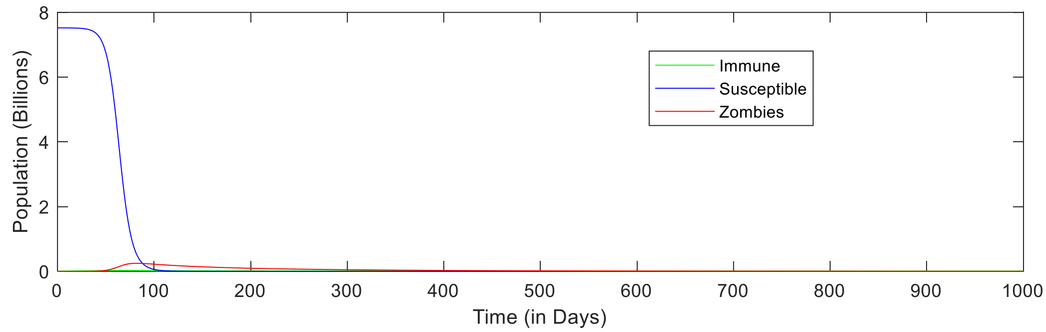
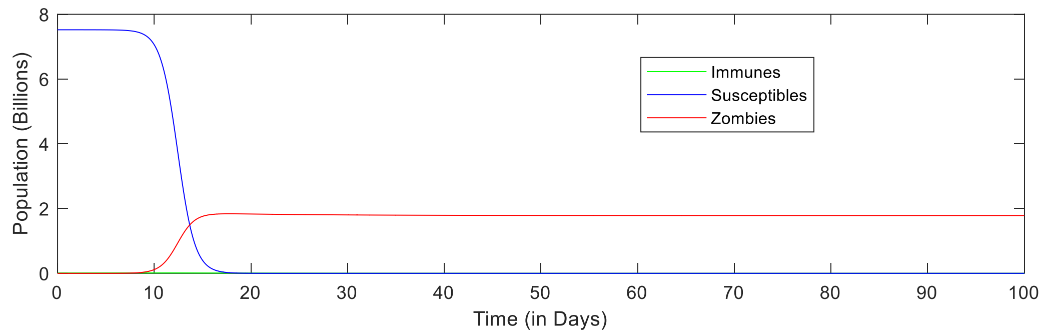
In the following case, we see that changing the outcomes of human-zombie interactions from 80% human victory, 18% zombification rate, and 2% human death from zombies rate to 45%/47.25%/2.75% produces the unique situation in which both humans and zombies have a stable equilibrium at 0. The rates signify that zombies and humans are essentially an equal match in a fight. In this situation, no population comes to control. This is the truest form of a "zombie apocalypse". Almost everyone dies within 100 days of the initial outbreak.



In the final scenario, zombies emerge as the sole form of life remaining. Again, by changing the outcome rates of human-zombie interactions, the outcome is vastly altered. The rates were this time changed to 40%/55%/5%. This situation could be likened to a fiercer, faster, harder to kill zombie. The graph below summarizes this outcome.



Evaluation

From the different scenarios posed in this paper, it is clear that humans can defeat the zombie outbreak given the right conditions. The key to human success hinges on their ability to effectively kill zombies when interacting with them. In order to survive the outbreak, it seems that the humans need to kill the zombies in at least 50% of their interactions. This goal seems realistic given the nature of weaponry and skill available to humans, including vast militaries. In conclusion, the most realistic assessment of this model is resounding success for humans. While the zombies claim the lives of many, the human population would not only meet but exceed its initial value within a few years after the outbreak.

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Appendix

Runge-Kutta-4 Code for Matlab

```
%write code for rk4 solver to solve zombie-human model
function [y,t] = rk4_n_dimensional(f,t0,T,y0,N);

h = (T-t0)/(N-1); %calculate and store the step size
t = linspace(t0,T,N); %A vector to store the time values
y = zeros(length(y0),N); %Initialize the Y vector with the same length as that of the 
initial value vector
y(:,1)=y0; %first column of Y matrix is the initial value

for i = 1:(N-1)
    k1=h*f(t(i),y(:,i));
    k2=h*f(t(i)+(.5*h),y(:,i)+(.5*k1));
    k3=h*f(t(i)+(.5*h),y(:,i)+(.5*k2));
    k4=h*f(t(i)+h,y(:,i)+k3);
    y(:,i+1)=y(:,i)+(1/6)*(k1+(2*k2)+(2*k3)+k4);
end
%end of rk4 portion of code
```

Code for Humans vs. Zombies model in Matlab

```
close all
%solver for zombie vs human model
%t is 1 day per time unit
bR = .00004781; %setting birth rate (there are 360,000 births per day, so 
360,000/7.53b per person per day)
Nd = .00002013; %setting rate of natural deaths (151,600 deaths per day so 151k/7.53b 
per person per day)
Zd = .02; %setting rate of deaths from zombies
Zb = .18; %setting rate of zombification (depends on how hungry they are)
Q  = .8; %setting rate of humans killing zombies (my zombies are really slow and dumb)
Im = .99; %setting vaccination success rate
Ki = .005; %setting vaccination mortality rate
Zo = .005; %setting vaccination zombification rate

ZvH = @(t,x)    [(-(Nd*x(1))-(Zd*x(1).*x(3))+(Im*bR.*(x(1)+x(2))));
                (-(Nd*x(2))-(Zd*x(2).*x(3))-(Zb*x(2).*x(3)));
                      ((Zb*(x(2).*x(3)))-(Q*(x(1)+x(2)).*x(3))
                     +((Zo*bR).*(x(1)+x(2))));
                     ((Nd*x(1))+(Nd*x(2))+(Zd*x(1).*x(3))+(Zd*x(2).*x(3))+(Q*(x(1)+x(2)
                     ).*x(3))+(Ki*(x(1)+x(2))))];

%initial conditions--7.53 billion people currently in the world
%say .1% are immune, or .00753 billion
%and 99.9% are susceptible, or 7.52247 billion
%no initial zombies or dead
[x,t] = rk4_n_dimensional(ZvH,0,10000,[.00753;7.52247;0;0],100000);

plot(t,x(1,:),'g')
hold on
plot(t,x(2,:),'b')
hold on
plot(t,x(3,:),'r')
```